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Topic: DIFFERENTIATION OF POLYNOMIALS

Introduction:

Differentiation is about finding **rates of change** of one quantity compared to another. Differentiation is used in analysis of finance and economics. One important application of differentiation is in the area of **optimization**, which means finding the condition for a maximum (or minimum) to occur. This is important in business (cost reduction, profit increase) and engineering (maximum strength, minimum cost.)

Video:

Simple Differentiation tutorial: https://www.youtube.com/watch?v=Ik1wFBLipiI

Outline:

- The general rule for differentiation
- Derivative of a constant
- Derivative of the nth power of an *x*
- Derivative of Constant product

There are many ways a question can ask you to differentiate:

- ***Differentiate*** *the function…*
- *Find* ***f '(x)***
- *Find* ***dy/dx***
- *Calculate the* ***rate of change*** *of…*
- *Find the* ***derivative*** *of…*
- *Calculate the* ***gradient of the tangent*** *to the curve…*

Summary:

The general rule for differentiation is:

$f(x) = ax^n \longrightarrow f'(x) = nax^{n-1}$

In other words: You bring the power down to the front to multiply and subtract 1 from the power.

Examples:

Q: *Differentiate* $y = x^5$

A:

$$\frac{dy}{dx} = 5x^4$$

Q: *Find the derivative of* $f(x) = 4x^3$

A:

$f'(x) = 12x^2$

A. Derivative of a Constant

$$\frac{dc}{dx} = 0$$

This means that if a quantity has a constant value, then the rate of change is zero

Example:

$$\frac{d}{dx}(6) = 0$$

B. Derivative of *n-th* power of *x*:

$$\frac{dc}{dx}x^n = nx^{n-1}$$

Example:

$$\frac{d}{dx}x^5 = 5x^4$$

C. Derivative of Constant product:

$$\frac{d}{dx}(cy) = c\frac{d}{dx}(y) = c\frac{dy}{dx}$$

Here, y is a function of x. So we find the derivative of that function first, then multiply by the constant.

Example:

$$if\ y = x^7,\ then\ \frac{dy}{dx} = \frac{d}{dx}(x^7) = 7x^6$$

Applying the new rule, we shall have:

$$\frac{d}{dx}(3y) = \frac{d}{dx}(3x^7)$$

$= 3\frac{d}{dx}(x^7)$

$= 3\frac{dy}{dx}$

$= (3)\,(7x^6)$

$= 21x^6$

Summary of Derivatives:

Constant	$\frac{dc}{dx} = 0$
n-th power of x:	$\frac{d}{dx}x^n = nx^{n-1}$
Constant product	$\frac{d}{dx}(cy) = c\frac{d}{dx}(y) = c\frac{dy}{dx}$

Example 1:

Find the derivative of $y = 7x^6$

Answer:

Using the rule:

$$\frac{d}{dx}(cy) = c\frac{d}{dx}(y)$$

We can take the 7 out to the front:

$$\frac{d}{dx}(7x^6) = 7\frac{d}{dx}(x^6)$$

And:

$$\frac{d}{dx}x^n = nx^{n-1}$$

Thus:

$$7\frac{d}{dx}(x^6) = 7 \times (6x^5)$$

$$= 42x^5$$

Note: This can be done in one step as follows:

$$\frac{dy}{dx} = 42x^5$$

Example 2:

Find the derivative of $y = 3x^5 - 1$

$$y = 3x^5 - 1$$

Now:

$$\frac{d}{dx}(3x^5) = 3 \times 5x^4 = 15x^4$$

And since: $\frac{dc}{dx} = 0$ it means that $\frac{d}{dx}(-1) = 0$

So:

$$\frac{dy}{dx} = \frac{d}{dx}(3x^5 - 1) = 15x^4$$

Example 3:

Find the derivative of:

$$y = 13x^4 - 6x^3 - x - 1$$

Now taking each term in turn:

$$\frac{d}{dx}(13x^4) = 52x^3 \ (using \ \frac{d}{dx}x^n = nx^{n-1})$$

$$\frac{d}{dx}(-6x^3) = -18x^2 \ (using \ \frac{d}{dx}x^n = nx^{n-1})$$

$$\frac{d}{dx}(-x) = -1 \ [since - x = (-(x^1) \ and \ so \ the \ derivative \ will \ be - (x^0) = -1]$$

$$\frac{d}{dx}(-1) = 0 \ (since \ \frac{dc}{dx} = 0)$$

So

$$\boldsymbol{\frac{dy}{dx} = 52x^3 - 18x^2 - 1}$$

Example 4:

Evaluate the derivative of $y = x^4 - 9x^2 - 5x$ *at the point (3, 15)*

Answer:

$y = x^4 - 9x^2 - 5x$

So:

$$\frac{dy}{dx} = 4x^3 - 18x - 5$$

At the point where x = 3, the derivative has value:

$$\frac{dy}{dx} = 4(3)^3 - 18(3) - 5$$

= 4x27 – 18x3 – 5

= 49.

This means that the slope of the curve $y = x^4 - 9x^2 - 5x$ *at 3 is 49.*

Exercises:

Find the derivatives:

1) $f(x) = x^9 + 8x^2 + x + 9$
2) $f(x) = 18x^2 + 9x^3 - 4x^7 - 91$

Answers:

1) $9x^3 + 16x + 1$
2) $36x + 27x^2 - 28x^6$
3) $240x^2 - 8x + 3$
4) $100x^{19} + 152x^{18} - 8x$

3) $g(x) = 80x^3 - 4x^2 + 3x + 7$

4) $g(x) = 5x^{20} + 8x^{19} - 4x^2 + 82$